

## Section Handout 6

*Based on a handout by Eric Roberts and Mehran Sahami*

### Problem One: The Coupon Collector's Problem

The *coupon collector's problem* is a famous probability problem with some interesting applications in computer science. Here's one way to frame it:

On average, how many times do you have to roll a six-sided die before every side comes up at least once?

Write a program that rolls a die until all sides come up at least once, then reports how many times the die had to be rolled before this happened.

### Problem Two: The Sieve of Eratosthenes

In the third century B.C., the Greek astronomer Eratosthenes developed an algorithm for finding all the prime numbers up to some upper limit  $N$ . To apply the algorithm, you start by writing down a list of the integers between 2 and  $N$ . For example, if  $N$  were 20, you would begin by writing down the following list:

2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

You then begin by circling the first number in the list, indicating that you have found a prime. You then go through the rest of the list and cross off every multiple of the value you have just circled, since none of those multiples can be prime. Thus, after executing the first step of the algorithm, you will have circled the number 2 and crossed off every multiple of two, as follows:

② 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ 15 ~~16~~ 17 ~~18~~ 19 ~~20~~

From here, you simply repeat the process by circling the first number in the list that is neither crossed off nor circled, and then crossing off its multiples. Eventually, every number in the list will either be circled or crossed out, as shown in this diagram:

② ③ ~~4~~ ⑤ ~~6~~ ⑦ ~~8~~ ~~9~~ ~~10~~ ⑪ ~~12~~ ⑬ ~~14~~ ~~15~~ ~~16~~ ⑰ ~~18~~ ⑱ ~~20~~

The circled numbers are the primes; the crossed-out numbers are composites. This algorithm for generating a list of primes is called the sieve of Eratosthenes. Write a program that uses the sieve of Eratosthenes to generate a list of all prime numbers between 2 and 1000.

### Problem Three: Inverting Colors

If you'll recall from our lecture on arrays, `GImage`s are internally represented on the computer as a two-dimensional array of pixels. Each pixel has a red, green, and blue component associated with them, and these values range from 0 (zero intensity) to 255 (maximum intensity). For example, pure red would have red value 255, green value 0, and blue value 0.

Given a `GImage`, we can construct the inverse of that `GImage` by flipping the intensity of each color channel. For example, if we have a pixel with red value 0, green value 255, and blue value 100, the inverse of that pixel would have red value 255, green value 0, and blue value 155. Visually, this represents taking the opposite of each of the colors in the image, so black pixels become white, green pixels become bright purple, etc.

Write a method

```
private GImage invertImage(GImage toInvert)
```

that accepts as input a `GImage` and produces a new `GImage` that's the inverse of the original `GImage`. As an example, here's a before-and-after comparison of the cover of Pink Floyd's "The Dark Side of the Moon" cover (would that make it the light side of the moon?)

